Tighter SONC bounds for polynomial optimization problems with bounded variable domains

Ksenia Bestuzheva, Ambros Gleixner, Helena Völker

The Combinatorial Optimization Workshop
CNRS Centre Paul Langevin, Aussois, France
January 09-13, 2023

Problem and Approach

Problem: find the global optimum of:

$$\min \sum_{\alpha \in \mathcal{A}(f)} f_\alpha \mathbf{x}^\alpha \text{ s.t. } \sum_{\alpha \in \mathcal{A}(g_i)} g_{i\alpha} \mathbf{x}^\alpha \geq 0 \text{ for all } i = 1 \ldots, m, \ \mathbf{x} \in \mathbb{R}^n.$$

Approach: look for $\max \gamma$ s.t. $L(\mathbf{x}, \mu) - \gamma \geq 0$ for all $\mathbf{x} \in \mathbb{R}^n$, where $L(\mathbf{x}, \mu)$ is the Lagrangian function.

Nonnegativity proof: via sums of nonnegative circuit polynomials (SONC) certificates. In a circuit polynomial $\sum_{\alpha \in V(f)} f_\alpha \mathbf{x}^\alpha + f_\beta \mathbf{x}^\beta$, exponents in $V(f)$ are affinely independent, and β can be written as their convex combination. Nonnegativity of a circuit polynomial can be decided immediately based on its coefficients and exponents.

SONC is an alternative to SOS and performs well for polynomials with high degrees and few terms.

Contribution and Results

Contributions:

- polynomial-bound constraints $x_i^{\alpha'} \leq \max\{|\ell_i|, |u_i|\}^{\alpha'}$ derived from variable bounds $[\ell_i, u_i]$ so that α' improves the structure of the exponent set of $L(\mathbf{x}, \mu)$,
- a branch-and-bound algorithm incorporating SONC relaxations implemented with SCIP and POEM.

Results:

- polynomial-bound constraints guarantee a finite SONC bound if all variables are bounded,
- SONC relaxations close 76.1-91.8% of the gap on 5 instances and 11.3% on 1 instance, out of 349,
- in the current implementation SONC relaxations are not competitive with LP relaxations within branch-and-bound.