SCIP Beyond 8.0

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The SCIP Optimization Suite

A toolbox for generating and solving MILPs, MINLPs, and CIPs:

- **SCIP** : MIP solver and CIP framework,
- **SoPlex**: LP solver,
- **PaPILO**: parallel presolver for integer and linear optimization,
- **ZIMPL**: mathematical programming language,
- **UG**: parallel framework for MIPs,
- **GCG**: generic branch-cut-and-price solver,
- **SCIP-SDP**: extension for solving MISDPs,
- **SCIP-Jack**: extenstion for solving Steiner tree and related problems.

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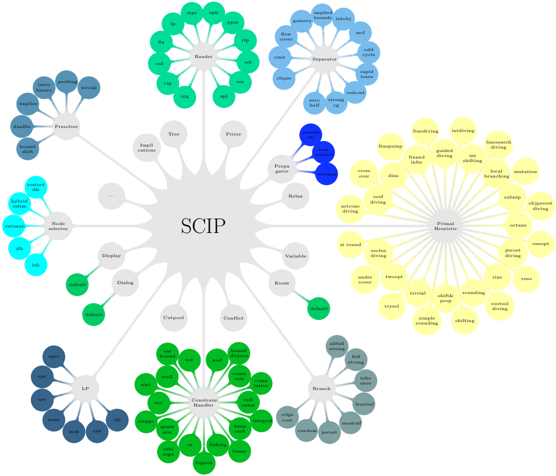
SCIP (Solving Constraint Integer Programs)

- Provides a **full-scale MILP and MINLP solver**,
- is **constraint based**,
- is a **branch-cut-and-price framework**,
- incorporates
  - **MILP features** (cutting planes, LP relaxation),
  - **MINLP features** (spatial branch-and-bound, OBBT)
  - **CP features** (domain propagation),
  - **SAT-solving features** (conflict analysis, restarts),
- has a modular structure via **plugins**,
- is **licensed under Apache 2.0**,
- and is **available in source-code** under `https://scipopt.org` !

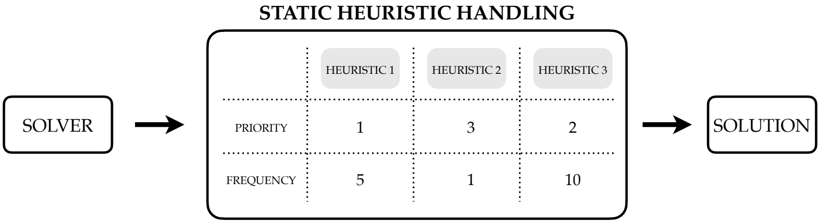


Overview of Recent Developments

- **Primal heuristics:**
  - Online learning for scheduling heuristics
  - Feasibility jump
  - Indicator diving
- **Cutting planes:**
  - Lift-and-project cuts
  - Lagromory cuts
  - Improved implicit product filtering for RLT cuts
  - Monoidal strengthening of intersection cuts for MIQCPs
- **Branching** via cutting plane selection
- Pseudo-Boolean **conflict analysis**
- Updates to the **exact solving** framework for MILPs
- Improvements to **symmetry handling**
- New and improved **interfaces**
  - SCIP will be able to call HiGHS (`https://highs.dev`) as an LP solver
  - New interface: Rust
  - Improvements to the Julia interface

Scheduling Primal Heuristics: Motivation

- MIP solving executes a broad range of primal heuristics for finding good solutions.
- The settings of heuristics are static with strict working limits.



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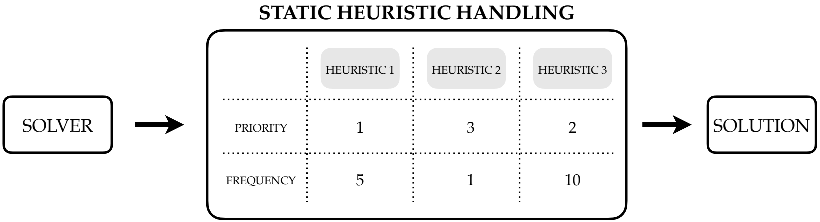


Question

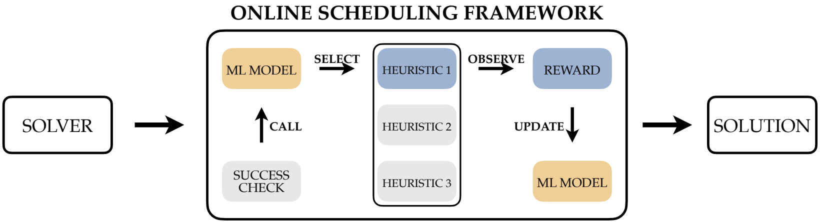
Static settings derived from heterogeneous benchmark test sets might not yield best performance since performance of heuristics is highly instance-dependent.

Idea

Make the execution of heuristics adaptive by learning which heuristics perform well for the current instance.

Scheduling Primal Heuristics: Online Learning

- A Chmiela, A Gleixner, P Lichocki, S Pokutta *Online Learning for Scheduling MIP Heuristics*
- Online scheduling framework manages (i) selection and (ii) working limits by learning from past observations.
- A novel reward function catches heuristics' impact on the solving process beyond simply finding new solutions.
- General framework enables us to schedule complex heuristics of different types simultaneously.



- Consistent node reductions over the MIPLIB 2017 Benchmark set.
- Speedup of 4% for instances that take at least 1000s to solve.

The Feasibility Jump Heuristic

B. Luteberget, G. Sartor *Feasibility Jump: an LP-free Lagrangian MIP heuristic*

- 1st place: MIP 2022 Computational Competition

Computational results on the MIPLIB benchmark:

- High success rate: Finds feasible solutions for over 30% of the instances
- Between 3 and 8% faster to the first feasible solution on average
- On average slightly slower

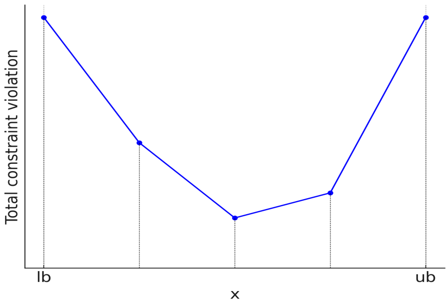
The Feasibility Jump Heuristic

It's a Lagrangian heuristic method: $\min c^T x$ s.t. $Ax \leq b \quad \rightarrow \quad \min \mathcal{L}(x, \lambda) = \lambda^\top (b - Ax)$

- Start with an incumbent vector x^*
- Choose a single variable
- "Jump" to the value that minimizes the weighted sum of constraint violations (taking integrality into account)
- The neighborhood, defined by scores, is updated after each jump "lazily"
- Score: decrease in total constraint violation

$$\max\{\lambda^\top (b - Ax), 0\} - \max\{\lambda^\top (b - Ax^*), 0\}$$

  (i.e. violation before the jump - violation after the jump)
- Update weights in the Lagrangian function



Indicator Diving Heuristic

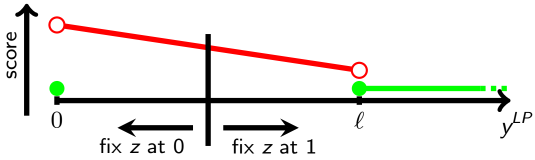
- A diving heuristic simulates a depth-first search.
  It alternates between tightening variable bounds and solving LP relaxations.

- Indicatordiving is a diving heuristic with focus on (unbounded) semi-continuous variables.

- Semi-cont. variables $y \in \{0\} \cup [\ell, u]$ with $u \in \mathbb{R}_+ \cup \{\infty\}$
  are modeled with a binary indicator variable: $z = 0 \rightarrow y = 0$
  $z = 1 \rightarrow y \geq \ell$

- During the diving process z is fixed depending on the LP solution value y^{LP} of the semi-cont. variable y:

$$y^{LP} < 0.5\,\ell \rightarrow z = 0$$
$$y^{LP} \geq 0.5\,\ell \rightarrow z = 1$$



Lift-and-project and Lagromory Cuts for MILPs

Lift-and-project cuts:

- Based on Bonami's 2012 work "On optimizing over lift-and-project closures"
- Goal: find cuts for the convex hull of a disjunction (e.g. branching)
- A trivial normalization constraint (NC) accounts for coefficient scaling
- NC $\rightarrow$ reduce the cut generating LP (CGLP) based on certain inferences
- Dualize the reduced CGLP $\rightarrow$ membership LP
- Solve membership LP, obtain dual information, and generate a cut

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Lagromory cuts:

- Based on Fischetti and Salvagnin's 2011 work "A relax-and-cut framework for Gomory mixed-integer cuts"
- In the root node consider Lagrangian dual problem, add GMI cuts as soft constraints
- GMI cuts 'tilt' the objective $\rightarrow$ explore nearby bases, add more GMI cuts
- Solve this problem iteratively by updating the Lagrangian multipliers
- Select cuts from the set of all thus generated GMI cuts to add to cut pool

Branching via Cutting Plane Selection: Motivation

Many cutting planes are derived from disjunctions. Most commonly from split disjunctions.

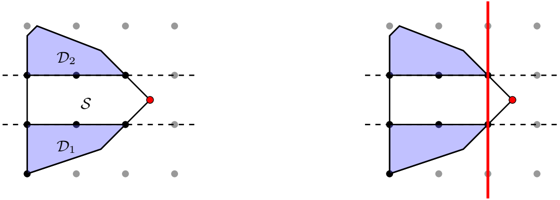

Figure: (Left) An example (simple) split. (Right) An example (simple) split cut.

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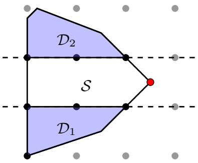

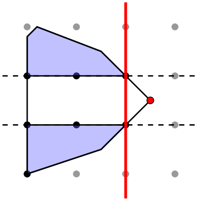

Figure: (Left) An example (simple) split. (Right) An example (simple) split cut.

Idea

Make branching decisions based on history of cut strength from similar disjunctions.

Branching via Cutting Plane Selection: Details

- Branching rule-1
  - Generate Gomori Mixed-Integer cuts from tableau rows corresponding to fractional basic variables.
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- Branching rule-2
  - Similar to above, but based on weak-GMI cuts.
- Branching rule-3
  - Generate GMI cuts similar to above.
  - Calculate the average cut strength.
  - Incorporate this as an additional metric into SCIP's default branching scoring function.
  - Select a branching candidate based on the cut with largest score.

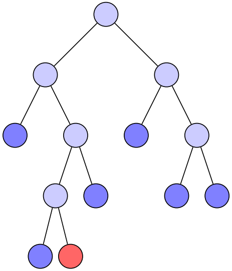
Results on MIPLIB 2017 benchmark:

- Rule 3 affects 67% of instances
- 4% reduction in mean time on affected instances

Conflict Analysis: Brief Introduction

When MIP solving reaches an infeasible subproblem, analyze the infeasibility to

- extract a shorter reason
- that prunes other parts of the tree
- and also helps in backtracking



- Generate a bound disjunction explaining the infeasibility similar to SAT solving.
  - Operates on clauses and not on the more expressive linear constraints
- Generate the Farkas constraint $(y^T A)x \geq y^T b$ for infeasible LPs.
  - May be dense with bad numerics

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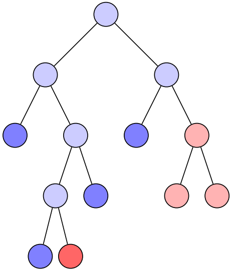

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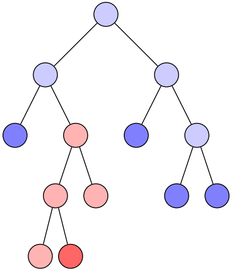

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Generalized Resolution Conflict Analysis

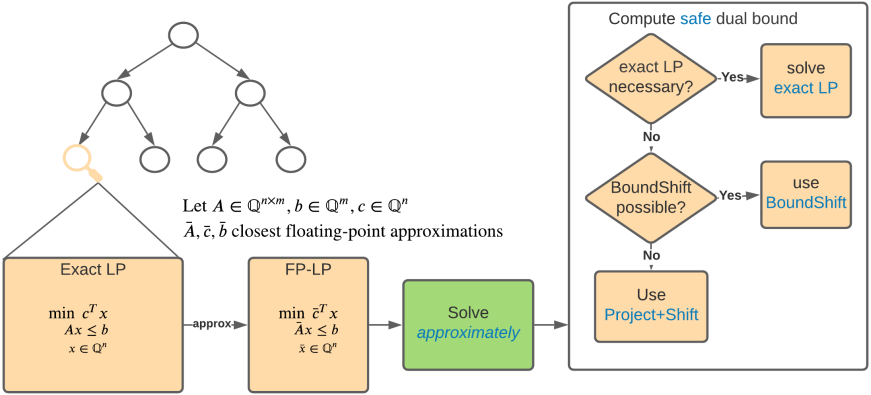
Goal: Given the infeasible system

$$\begin{cases} a^\top x \geq a_o & \text{(reason: for propagating } x_i \geq \alpha) \\ b^\top x \geq b_o & \text{(conflict: infeasible for } x_i \geq \alpha) \\ x \in [\ell', u'] \subset [\ell, u]. \end{cases}$$

Can we find a single constraint that proves the infeasibility?

- G Mexi, T Berthold, A Gleixner, J Nordstroem *Improving Conflict Analysis in MIP Solvers by Pseudo-Boolean Reasoning*
- Applicable to pure binary constraints
- "massage" reason constraint until it propagates x_i tightly.
  - Weakening: Set variables at global bounds **and**
  - Stengthening: MIR, CG, Coef. Tightening
- "Resolve" x_i (add the two constraints so that x_i disappears)

Exact MILP Solving: Hybrid Branch-and-Bound



Implemented in SCIP: more details in Cook, Koch, Steffy, Wolter 2013.

Uses floating-point + directed rounding + rational arithmetic.

Exact MILP Solving: New Exact SCIP Features

Eifler, Gleixner 2021 & 2022:

- thorough revision of hybrid-precision branch and bound
- integrate SoPlex as exact LP solver (Gleixner, Steffy 2019 & 2020)
- addition of rational presolving (Gleixner, Gottwald, Hoen 2023)
- addition of primal heuristics
- output of VIPR certificates (Cheung, Gleixner, Steffy 2017)

Eifler, Gleixner 2023 (preprint available)

- safe, verified generation of Gomory mixed-integer cuts

Published soon:

- domain propagation + conflict analysis (Borst, Eifler, Gleixner)
- precision boosting + iterative refinement in exact LP (Eifler, Gleixner, Thouvenin)

Symmetries in MIPs

Symmetries of a MIP

$$\max\{c^T x \ : \ Ax \leq b, \ x \in \mathbb{Z}^n\}$$

are bijections $f\colon \mathbb{R}^n \to \mathbb{R}^n$ such that $x \in \mathbb{R}^n$ is feasible iff $f(x)$ is feasible and both have the same objective value.

Issue Branch-and-bound trees become unnecessarily large since symmetric subproblems are explored multiple times.

$$\begin{aligned} \max x_1 + x_2 & \\ x_1 \qquad + 2x_3 &\leq 3 \\ x_2 + 2x_3 &\leq 3 \end{aligned}$$

SCIP's Symmetry Handling Tools

Tools in SCIP 8.0

- automatic symmetry detection
- symmetry handling constraints (orbitopes, orbisacks, symresacks, SST cuts)
- propagation algorithms (orbital fixing)

Issue: Constraint-based and propagation-based methods can not be combined.

Latest Symmetry Handling Changes

- completely revised symmetry handling framework that allows to combine constraints and propagation algorithms.
- at the time of merging, the new framework improves on the old framework by 5.9% (25.4% on instances running at least 1000s).
- interface to graph automorphims code sassy to accelerate symmetry detection

Thank you!